12th CBSE(SAT-1)
(SESSION : 2014)
SUBJECT : MATHS(गणित)
SOLUTION (हल)

1. $R = \{(x, y) : x + 2y = 8\}$

$x + 2y = 8$

Put $x = 2$, $2 + 2y = 8$, $y = 3$ (2, 3)

Put $x = 4$, $4 + 2 \times y = 8$, $y = 2$ (4, 2)

Put $x = 6$, $6 + 2y = 8$, $y = 1$ (6, 1)

$\therefore$ range if {1, 2, 3}

2. $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ if $xy < 1$

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$

$\frac{x+y}{1-xy} = \tan\frac{\pi}{4}$

$x + y = 1 - xy$

$x + y + xy = 1$

3. Given

$A^2 = A$

$7A - (I + A)^3 = 7A - [I^3 + A^3 + 3I.A. (A + I)]$

$= 7A - [I + A. A + 3 I. A. A. + 3I. A.I.]$

$= 7A - I - A. -3A - 3A$

$= - I$

4. $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$

$x - y = - 1$

$2x - y = 0$

$- x = - 1$

$x = 1$

$y = 2$

Then $x + y = 3$

5. $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$12x + 14 = 32 - 42$

$12x + 14 = -10$

$12x = - 24$

$x = - 2$

6. $f(x) = \int_0^x t\sin t\,dt$

$f'(x) = [t\sin t]_0^x$

$= x \sin x$

7. $\int_2^4 \frac{x}{x^2+1}dx$ multiply and divide by 2

$\int_2^4 \frac{2x}{2(x^2+1)}dx$

$\frac{1}{2}\left[\log(x^2+1)\right]_2^4$

$\frac{1}{2}\left[\log 17 - \log 5\right]$

$\frac{1}{2}\log\frac{17}{5}$

8. $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel

so $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

$-3p = 1$

$p = -1/3$

9. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = ?$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(4-1) - \hat{j}(-2-3) + \hat{k}(-1-6)$

$= = 3\hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{a}\cdot(\vec{b}\times\vec{c}) = (2\hat{i} + \hat{j} + 3\hat{k}).(3\hat{i} + 5\hat{j} - 7\hat{k})$

$= 6 + 5 - 21$

$= -10$

10. $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$

$\frac{x-3}{-5} = \frac{y-(-4)}{7} = \frac{z-3}{2}$

we know that the equation of line

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = (x_1\hat{i} + y_1\hat{j} - z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} - c\hat{k})$

$\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$

11. $f : R \rightarrow R$, $f(x) = x^2 + 2$, $g : R \rightarrow R$, $g(x) = \frac{x}{x-1}$, $x \neq 1$

fog (x) = f [g(x)]

$fog(x) = \left(\frac{x}{x-1}\right)^2 + 2$

fog(2) = 4 + 2 = 6

gof (x) = g[f(x)]

$gof(x) = \frac{x^2+2}{x^2+1}$

$gof(-3) = \frac{9+2}{9+1} = \frac{11}{10}$

12. $x = \cos 2\theta$ ..(1)

$= \tan^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right\}$

$= \tan^{-1}\left\{\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right\}$

$= \tan^{-1}\left\{\frac{1-\tan\theta}{1+\tan\theta}\right\}$

$= \tan^{-1}\tan\left(\frac{\pi}{4}-0\right)$

$= \frac{\pi}{4} - \theta$

$= \frac{\pi}{2} - \frac{1}{2}\cos^{-1} x$

OR

$\tan^{-1}\frac{x-2}{x-4} + \tan^{-1}\frac{x+2}{x+4} = \frac{\pi}{4}$

$\tan^{-1}\left[\frac{\frac{x-2}{x-4}+\frac{x+2}{x+4}}{1-\frac{x-2}{x-4}\times\frac{x+2}{x+4}}\right] = \frac{\pi}{4}$

$\frac{(x-2)(x+4)+(x+2)(x-4)}{(x^2-16)-(x^2-4)} = 1$

$x^2 + 2x - 8 + x^2 - 2x - 8 = (x^2 - 16) - (x^2 - 4)$

$2x^2 - 16 = -16 + 4$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

13. Given

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

L.H.S.

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

$$\begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$$

take common elements

$$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix}$$ Two coloumn are identical

$c_2 \to c_2 - c_1$
$c_3 \to c_3 - c_1$

$$x^3 \begin{vmatrix} 1 & 0 & 0 \\ 5 & -1 & -3 \\ 10 & -2 & -7 \end{vmatrix} + 0$$

$x^3 (7 - 6) = x^3$

14. $x = ae^{\theta} (\sin\theta - \cos\theta)$
$y = ae^{\theta} (\sin\theta + \cos\theta)$

if $q = \frac{\pi}{4}$

$$\frac{dx}{d\theta} = ae^{\theta} [\cos\theta + \sin\theta] + (\sin\theta - \cos\theta)\, ae^{\theta} = ae^{\theta} (2 \sin \theta)$$

$$\frac{dy}{d\theta} = ae^{\theta} [\cos\theta - \sin\theta] + (\sin\theta + \cos\theta)\, ae^{\theta} = ae^{\theta} (2 \cos \theta)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{ae^{\theta} 2\cos\theta}{ae^{\theta} 2\sin\theta} = \cot\theta$$

$$\frac{dy}{dx} = \cot\theta$$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = \cot \frac{\pi}{4} = 1$$

15. $y = Pe^{ax} + Qe^{bx}$
show that

$$\frac{d^2y}{dx^2} - (a + b) \frac{dy}{dx} + aby = 0$$

$$\frac{dy}{dx} = Pae^{ax} + Qbe^{bx}$$

$\frac{d^2y}{dx^2} = Pa^2e^{ax} + Qb^2e^{bx}$

L.H.S.

$\frac{d^2y}{dx^2} - (a + b)\frac{dy}{dx} + aby$

$(Pa^2e^{ax} + Qb^2e^{bx}) - (a + b)(Pae^{ax} + Qbe^{bx}) + aby = 0$

$Pa^2e^{ax} + Qb^2e^{bx} - Pa^2e^{ax} - Qabe^{bx} - Pabe^{ax} - Qb^2e^{bx} + aby = 0$

$-ab\,|Qe^{bx} + Pe^{ax}| + aby$

$-aby + aby = 0$

16. $y = [x(x-2)]^2$

$\frac{dy}{dx} = 2(x(x-2))[(x-2) + x]$

$= 2[x^2 - 2x][2x - 2]$

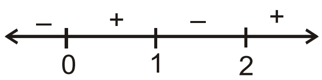
$= 4[(x^2 - 2x)(x - 1)]$

for increasing faction

$\frac{dy}{dx} > 0$

$4(x(x-2)(x-1)) > 0$

$x > 0 \quad x > 2 \quad x > 1$

f(x) is increasing on 

$(0, 1) \cup (2, \infty)$

17. $I = \int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}dx$ ..(1)

we know that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi} \frac{4(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$ ..(2)

by adding equation (1) and equation (2)

$2I = \int_0^{\pi} \frac{4\pi\sin x}{1+\cos^2 x}dx$ Let cos x = t

$-\sin x\,dx = dt$ when x = 0, t = 1

$x = \pi$, t = – 1

$2I = \int_1^{-1} \frac{4\pi(-dt)}{1+t^2}$

$2I = -4\pi\,[\tan^{-1} t]^{-1}_{+1}$

$2I = -4\pi\,[-\frac{\pi}{4} - \frac{\pi}{4}]$

$2I = 4p \times \frac{2\pi}{4} = 2\pi^2$

$I = \pi^2$

OR

$$\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

$$\int \frac{\frac{1}{2}(2x+5-5)+2}{\sqrt{x^2+5x+6}}dx$$

$$\frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x^2+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

if $x^2 + 5x + 6 = t$

$(2x + 5)\ dx = dt$

$$\frac{1}{2}\int \frac{dt}{\sqrt{t}} - \frac{1}{2}\log|(x + 5/2) + \sqrt{(x+5/2)^2-(1/2)^2}$$

$$\frac{1}{2}2\sqrt{t} - \frac{1}{2}\log|(x + 5/2) + \sqrt{(x+5/2)^2-\left(\frac{1}{2}\right)^2} + C$$

$$\frac{1}{2}\sqrt{4^2+5x=6} - \frac{1}{2}\log|(x + 5/2) + \sqrt{x^2+5x+6}| + C$$

$$\sqrt{x^2+5x+6} - \frac{1}{2}\log|(x + 5/2) + \sqrt{x^2+5x+6} + C$$

18. $\frac{dy}{dx} = 1 + x + y + xy$

$$\frac{dy}{dx} = 1 + x + y[1 + x]$$

$$\frac{dy}{dx} = (1 + x)(1 + y)$$

$$\frac{dy}{1+y} = (1 + x)\ dx$$

intergrating both sides

$$\int \frac{dy}{1+y} = \int (1+x)dx$$

$$\log(1 + y) = x + \frac{x^2}{2} + C$$

Now put x = 1 and y = 0

$$\log 1 = 1 + \frac{1}{2} + C$$

$$0 = \frac{3}{2} + C$$

$$C = \frac{-3}{2}$$

$$\log(1 + y) = x + \frac{x^2}{2} - \frac{3}{2}$$

19. $(1 + x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1}x}$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$$

Solution of equation is given by

$$y.(\text{I.F.}) = \int \frac{e^{\tan^{-1}x}}{1+x^2}.(\text{I.F.})\,dx$$

$$y.e^{\tan^{-1}x} = \int \frac{\left(e^{\tan^{-1}x}\right)^2}{1+x^2}.dx$$

put $e^{\tan^{-1}x} = t$

$$\frac{e^{\tan^{-1}x}}{1+x^2}.\,dx = dt$$

$$y.e^{\tan^{-1}x} = \int t.dt$$

$$y.e^{\tan^{-1}x} = \frac{t^2}{2} + C$$

$$y.e^{\tan^{-1}x} = \frac{(e^{\tan^{-1}x})^2}{2} + C$$

20. $\overrightarrow{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$

$\overrightarrow{OB} = -\hat{j} - \hat{k}$

$\overrightarrow{OC} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$\overrightarrow{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

Now these four points will be coplaner if

$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$ ...(1)

$\overrightarrow{AB} = (-\hat{j} - \hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$

$\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = (3\hat{i} + 9\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$

$\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$\overrightarrow{AD} = -(\hat{i} + 4\hat{j} + 4\hat{k}) - (4\hat{i} + 5\hat{j} + \hat{k})$

$\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$

by equation (1) $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]$

$$= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$= -4\,(12 + 3) + 6\,(-3 + 24) - 2(1 + 32)$

$= -60 + 126 - 66$

$= 0$

So All four points are coplanar

21. Equation of straight line passing through (2, –1, 3) is

$$\frac{x-2}{a} = \frac{y+1}{b} = \frac{z-3}{c} \qquad ...(1)$$

lines (1) is perpendicular to two lines

$\therefore$ 2a – 2b + c = 0

a + 2b + 2c = 0

$$\frac{a}{-4-2} = \frac{-b}{4-1} = \frac{c}{4+2}$$

$$\frac{a}{-6} = \frac{b}{-3} = \frac{c}{6}$$

$\therefore$ Equation of plane is

cartesian form $\frac{x-2}{-6} = \frac{y+1}{-3} = \frac{z-3}{6}$

Vector form $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(-6\hat{i} - 3\hat{j} + 6\hat{k})$

22. P(success) = 3P(fails)

P(success) + P(fails) = 1

3P(fails) + P(fails) = 1

$\therefore$ P(fails) = $\frac{1}{4}$

P(success) = $\frac{3}{4}$

$P(x \geq 3) = P(x = 3) + P(x = 4) + P(x - 5)$

$= 5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + 5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^1 + 5C_5\left(\frac{3}{4}\right)^5$

$= 10 \times \frac{27}{64} \times \frac{1}{16} + 5 \times \frac{81}{256} \times \frac{1}{4} + \frac{243}{1024}$

$= \frac{270}{1024} + \frac{405}{1024} + \frac{243}{1024}$

$= \frac{918}{1024} = \frac{459}{512}$

23. $\begin{bmatrix} \text{sincerity} & \text{truthfulness} & \text{helpfulness} \\ 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$

AX = B

$X = A^{-1}B$

$A^{-1} = \frac{\text{adj}A}{|A|}$

|A| = 3(1 – 3) – 2 (4 – 3) + 1 (4 – 1)

|A| = –5

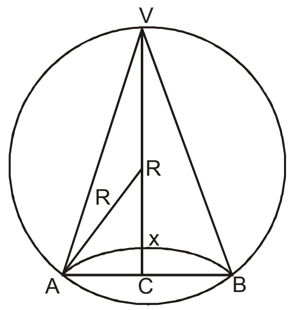
adj A = $\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

$X = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$

$$= -\frac{1}{5}\begin{bmatrix}-1000\\-1500\\-2000\end{bmatrix} = \begin{bmatrix}200\\300\\400\end{bmatrix}$$

X = 200 , Y = 300, Z = 400

24.



$AC = \sqrt{R^2 - x^2}$

VC = VO + OC = R + x

$V = \frac{1}{3}\pi(AC)^2 (VC)$

$V = \frac{1}{3}\pi (R^2 - x^2) (R + x)$

$V = \frac{1}{3}\pi (R^3 + R^2x - x^2R - x^3)$

$\frac{dv}{dx} = \frac{1}{3}\pi [R^2 - 2Rx - 3x^2]$

$\frac{dv}{dx} = 0$

$R^2 - 2Rx - 3x^2 = 0$

$(R - 3x) (R + x) = 0$

$x = \frac{R}{3}$

$\frac{d^2v}{dx^2} = \frac{1}{3}\pi (-2R - 6x)$

at $x = \frac{R}{3}$

$\frac{d^2v}{dx^2} = \frac{1}{3} p [-2R - 6 \times \frac{R}{3}] < 0$

V is maximum

Altitude is $VC = R + \frac{R}{3} = \frac{4R}{3}$

$V = \frac{1}{3}(R^2 - x^2) (R + x)$

$= \frac{1}{3}(R^2 - \frac{R^2}{9}) (R + \frac{R}{3}) = \frac{32\pi R^3}{81}$

$V = \frac{8}{27}\left[\frac{4}{3}\pi R^3\right] = \frac{8}{27}$ volume of sphere

25. $\int \frac{dx}{\cos^4 x + \sin^4 x}$

Divide by $\cos^4 x$

$= \int \frac{\sec^4 x}{1+\tan^4 x} dx$

$= \int \frac{\sec^2 x.\sec^2 x}{1+\tan^4 x} dx$

$= \int \frac{(1+\tan^2 x).\sec^2 x}{1+\tan^4 x} dx$

Put tan x = t

$\sec^2 x dx = dt$

$= \int \frac{1+t^2}{1+t^4} dt$

Divide by t^2

$= \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} dt$

$= \int \frac{1+\frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + (\sqrt{2})^2} dt$

let $t - \frac{1}{t} = u$

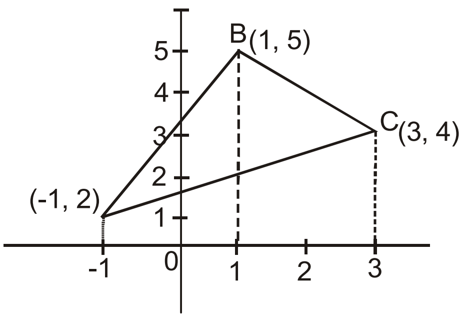
$= \left(1+\frac{1}{t^2}\right) dt = du$

$= \int \frac{du}{u^2+(\sqrt{2})^2}$

$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{u}{\sqrt{2}} + C$

$= \frac{1}{\sqrt{2}} \tan^{-1} \left[\frac{\tan x - \cot x}{\sqrt{2}}\right] + C$

26.



Equation line AB

A(–1, 2) ; B(1, 5)

$y - 2 = \frac{5-2}{1+1}(x + 1)$

$y - 2 = \frac{3}{2}(x + 1)$

$2y - 4 = 3x + 3$

$2y = 3x + 7$

$y = \frac{3x+7}{2}$ ..(1)

equation line BC

B(1, 5) ; C(3, 4)

$y - 5 = \frac{4-5}{3-1}(x - 1)$

$y - 5 = \frac{-1}{2}(x - 1)$

$2y - 10 = -x + 1$

$y = \frac{-x+11}{2}$ ..(2)

equation of line AC

A(–1, 2) ; C(3, 4)

$y - 2 = \frac{4-2}{4}(x + 1)$

$4y - 8 = 2x + 2$

$y = \frac{2x+10}{4} = \frac{x+5}{2}$

area of $\Delta ABC = \int_{-1}^{1}\frac{3x+7}{2}dx + \int_{1}^{3}\frac{-x+11}{2}dx - \int_{-1}^{3}\frac{x+5}{2}dx$

$= \frac{1}{2}\left[\frac{3x^2}{2}+7x\right]_{-1}^{1} + \frac{1}{2}\left[\frac{-x^2}{2}+\frac{1}{x}\right]_{1}^{3} - \frac{1}{2}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$

$= \frac{1}{2}\left[\left(\frac{3}{2}+7\right)-\left(\frac{3}{2}-7\right)+\left(\frac{9}{2}+33\right)-\left(-\frac{1}{2}+11\right)\right] - \frac{1}{2}\left[\left(\frac{9}{2}+15\right)-\left(\frac{1}{2}-5\right)\right]$

$= \frac{1}{2}\left[14-\frac{9}{2}+33+\frac{1}{2}-11\right] - \frac{1}{2}[4 + 20]$

$= \frac{1}{2}[32] - \frac{1}{2}[24]$

$= 16 - 12$

= 4 square units

27. Equation of plane passing through the intersection of planes is

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$

$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 + 4\lambda) - 1 - 5\lambda = 0$ ... (1)

plane (1) is prep. to the plane

$x - y + z = 0$

$\therefore 1(1 + 2\lambda) - 1(1 + 3\lambda) + 1(1 + 4\lambda) = 0$

$1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$

$3\lambda + 1 = 0$

$\lambda = -\frac{1}{3}$

So, equation of plane is

$$(x + y + z - 1) - \frac{1}{3}(2x + 3y + 4z - 5) = 0$$

$3x + 3y + 3z - 3 - 2x - 3y - 4z + 5 = 0$
$x - z + 2 = 0$
Distance of plane formation

$$= \frac{|1-1+2|}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ unit}$$

OR

Line

$$\frac{x-2}{3} = \frac{y+4}{4} = \frac{z-2}{2} = l$$

$x = 3\lambda + 2$
$y = 4\lambda - 4$
$z = 2\lambda + 2$
let the intersection point of line and plane is
$(3\lambda + 2, 4\lambda - 4, 2\lambda + 2)$
Now this point will lies on plane $x - 2y + z = 0$
$\therefore (3\lambda + 2) - 2(4\lambda - 4) + (2\lambda + 2) = 0$
$3\lambda + 2 - 8\lambda + 8 + 2\lambda + 2 = 0$
$-3\lambda + 12 = 0$
$-3\lambda = -12$
$\lambda = 4$
$\therefore$ point of intersection is : (14, 12, 10)
Distance between pont (2, 12, 5)

$$\text{Distance} = \sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2}$$

$$= \sqrt{144 + 0 + 25}$$

$$= \sqrt{169} = 13 \text{ unit}$$

28.

	A	B
Fabricating	9	12
Finishing	1	3

$9x + 12y \leq 180$
$x + 3y \leq 30$
constraints
max. $Z = 80x + 120y$
$9x + 12y \leq 180$
$3|3x + 4y| \leq 180$
$3x + 4y \leq 60$
$x + 3y \leq 30$
$x \geq 0$
$y \geq 0$
$3x + 4y \leq 6\ 0$

x = 0 ; y = 15
y = 0 ; x = 20
(0, 15) (20, 0)
$x + 3y \leq 30$
if x = 0 ; y = 10
y = 0 x = 30
(0, 10) (30, 0)
3x + 4y = 60 ..(1)
x + 3y = 30 ..(2)
equation (2) multiply by 3
3x + 4y = 60
3x + 9y = 90

– 5y = – 30
y = 6 ..(3)
x + 18 = 30
x = 12 .. (4)
(12, 6) intersecting point
z = 30x + 120 y (0, 10), (12, 6), (20, 0)
If P_1 (0, 10)
z = 1200
If P_z (12, 6)
$z = 80 \times 12 + 120 \times 6$
= 960 + 720
= 1680
If P_3 (20, 0)
z = 1600
maximum profit per week = 1680

29. E_1 = Two headed coins
E_2 = Biased coin that comes up head 75% times
E_3 = Biased coin that comes up head 40% of times
A = Head

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$$

$$P(A/E_1) = \frac{2}{2} = \frac{20}{20} \quad P(A/E_2) = \frac{75}{100} = \frac{3}{4} = \frac{15}{20}$$

$$P(A/E_3) = \frac{60}{100} = \frac{12}{20}$$

$$\therefore P(E_1/A) = \frac{P(E_1)\times P(A/E_1)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{20}{20}}{\frac{1}{3}\times\frac{20}{20}+\frac{1}{3}\times\frac{15}{20}+\frac{1}{3}\times\frac{12}{20}}$$

$$= \frac{20}{20+15+12} = \frac{20}{47}$$

OR

X = larger of two numbers

$P(X = 2) = 2 \times \frac{1}{6} \times \frac{1}{5} = \frac{2}{30}$

$P(X = 3) = 4 \times \frac{1}{6} \times \frac{1}{5} = \frac{4}{30}$

$P(X = 4) = 6 \times \frac{1}{6} \times \frac{1}{5} = \frac{6}{30}$

$P(X = 5) = 8 \times \frac{1}{6} \times \frac{1}{5} = \frac{8}{30}$

$P(X = 6) = 10 \times \frac{1}{6} \times \frac{1}{5} = \frac{10}{30}$

Probability distribution of X

x	2	3	4	5	6
P(x)	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

Mean

Xi	Pi	PiXi
2	2/30	4/30
3	4/30	12/30
4	6/30	24/30
5	8/30	40/30
6	10/30	60/30

Mean = $\sum PiXi$

$\sum PiXi = \frac{140}{30} = \frac{14}{3}$