19. $D = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

Taking a, b and c common from c_1, c_2 and c_3 respectively

$$\Delta = abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & 1+\frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

Applying $c_1 \rightarrow c_1 + c_{\backslash 2} + c_3$

$$\Delta = abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & 1+\frac{1}{c} \end{vmatrix}$$

$\therefore$ taking $(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c})$ common from c_1

$$\Delta = abc\ (1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \begin{vmatrix} 1 & 1/b & 1/c \\ 1 & 1+1/b & 1/c \\ 1 & 1/b & 1+1/c \end{vmatrix}$$

$$\Delta = abc\ (1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \begin{vmatrix} 1 & 1/b & 1/c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$ [$\therefore$ applying $R_2 \rightarrow R_2 - R_1$ $R_3 \rightarrow R_3 - R_1$]

$$\Delta = abc\ (1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \times 1 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$ [expanding along c_1]

$$\Delta = abc\ (1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c})$$

$$\Delta = abc \left(\frac{abc + bc + ca + ab}{abc}\right)$$

$\Delta = abc + bc + ca + ab$

20. $x = 3\cos t - 2\cos^3 t$

$\frac{dx}{dt} = -3\sin t + 6\cos^2 t.\ \sin t$

$$\frac{dx}{dt} = -3\sin t\,(1 - 2\cos^2 t)$$

$$\frac{dx}{dt} = 3\sin t.\cos 2t \qquad ..(1)$$

$y = 3\sin t - 2\sin^3 t$

$$\frac{dy}{dt} = 3\cos t - 6\sin^2 t\cos t$$

$= 3\cos t\,(1 - 2\sin^2 t)$

$$\frac{dy}{dt} = 3\cos t\,.\,\cos 2t \qquad ..(2)$$

Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos t.\cos 2t}{3\sin t.\cos 2t}$

$$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = 1$$

21. $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$$\frac{dy}{dx} = e^{(3x+4y)}$$

$$\frac{dy}{dx} = e^{3x}.\,e^{4y}$$

$e^{-4y}.\,dy = e^{3x}.dx$

Intergrating both sides

$$\int e^{-4y}.dy = \int e^{3x}.dx$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

Now given when x = 0, y = 0

$$\frac{e^0}{-4} = \frac{e^0}{3} + C$$

$$\frac{-1}{4} = \frac{1}{3} + C \Rightarrow C = \frac{-7}{12}$$

So $\frac{-e^{-4y}}{4} = \frac{-e^{3x}}{3} - \frac{7}{12}$

22. Given lines

$l_1 : \frac{1-x}{3} = \frac{7y-14}{P} = \frac{z-3}{2}$

$$\frac{x-1}{-3} = \frac{y-2}{P/7} = \frac{z-3}{2} \qquad ..(1)$$

$l_2 : \frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5}$

$$\frac{x-1}{-3P/7} = \frac{y-5}{1} = \frac{z-6}{-5} \qquad ..(2)$$

Now $\therefore$ lines (1) and (2) are perpendicular to each other

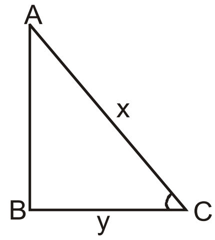
So $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$-3 \times \left(\frac{-3P}{7}\right) + \frac{P}{7} \times 1 + 2(-5) = 0$$

$$\frac{10P}{7} = 10 \qquad \Rightarrow P = 7$$

Now equation of the line passing through a point (3, 2, –4) and parallel to line l_1 is given by

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

28. 

Given :

$x + y = K$

$x = K - y$

$AB = \sqrt{x^2 - y^2}$

$A = \frac{1}{2}\ AB \times BC$

$A = \frac{1}{2}\ \sqrt{x^2 - y^2}\ .\ y$

$A^2 = \frac{1}{4}\ (x^2 - y^2)\ y^2$

$A^2 = \frac{1}{4}\ [(K - y)^2 - y^2]\ y^2$

$A^2 = \frac{1}{4}\ [K^2 - 2Ky + y^2 - y^2]\ y^2$

$A^2 = \frac{1}{4}\ [K^2Y^2 - 2Ky^3]$

Diff. w.r.t y

$2A\ \frac{dA}{dy} = \frac{1}{4}\ [K^2 2y - 6Ky^2]$

For the critical point

$\frac{dA}{dy} = 0$

$2K^2y - 6Ky^2 = 0$

$6Ky^2 = 2K^2y$

$y = \frac{K}{3}$

$x = K - y$

$= K - \frac{K}{3} = \frac{2K}{3}$

$\cos q = \frac{y}{x} = \frac{K/3}{2K/3} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$2A \frac{dA}{dy} = \frac{1}{4} [K^2y - 6Ky^2]$

diff. w.r.t y

$2A\frac{d^2A}{dy^2} + 2\left(\frac{dA}{dy}\right)^2 = \frac{1}{4}[K^2 - 12\ Ky]$

when $\frac{dA}{dy} = 0$

$2A\frac{d^2A}{dy^2} = \frac{1}{4} [K^2 - 12 \times K \times \frac{K}{3}]$

$= \frac{1}{4} (-3K^2) < 0$

$\therefore$ Area is maximum when angle between hypotenute and side of triangle is 60°

29. $\int \frac{dx}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}$

$= \frac{dx}{(\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x}$

$= \int \frac{dx}{1 - \sin^2 x \cos^2}$

Divide by $\cos^4 x$

$= \int \frac{\sec^4 x}{\sec^4 x - \tan^2 x} dx$

$= \int \frac{\sec^2 x.\sec^2 x}{(\sec^2 x)^2 - \tan^2 x} dx$

$= \int \frac{(1+\tan^2 x).\sec^2 x}{(1+\tan^2 x)^2 - \tan^2 x} dx$

Let tan x = t

$\sec^2 x\ dx = dt$

$= \int \frac{1+t^2}{(1+t^2)^2 - t^2} dt$

$= \int \frac{1+t^2}{1+t^4+2t^{-2}-t^2} dt$

$= \int \frac{1+t^2}{t^4+t^2+1} dt$

Divide by t^2

$\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}+1} dt$

Let $t - \frac{1}{t} = u$

$= \left(1+\frac{1}{t^2}\right)$ dt = du

$= \int \frac{du}{u^2+3}$

$= \int \frac{du}{u^2+\left(\sqrt{3}\right)^2}$

$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + C$

$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{\tan x - \cot x}{\sqrt{3}} + C$